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Radical Ignorance In Individual Decision Making:

Assessing Austrian Subjectivism

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Abstract - In recent years most Austrian developments have argued for a definitive shift away from the search for equilibrium constructs and in favor of the analysis of those institutions which favor ordered outcomes of the market process. These developments imply an inevitable withdrawal from methodological individualism and are based on the contention that a probabilistic approach to subjective decision making is flawed. Our aim in this paper is to point out that a withdrawal from equilibrium theorizing is not justified by the inability of pure economic theory to deal with radical ignorance. We argue that the kind of formal representation of decision making under uncertainty one finds in recent developments in microeconomic theory, namely the non-additive approach to subjectively probable assessments, recognizes as a starting point for research the view that ignorance is an inherent feature of every decision regarding future events. In this, it resembles the Shackleian assertion that the future is the unpredictable consequence of creative choices made by individual agents. A critical, but positive, attitude towards recent attempts to formalize radical ignorance suggests that the Austrian tradition may actually influence future research rather than merely constitute an optional supplement to it.

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Introduction

Most representatives of the radical subjectivist approach, notably neo-Austrians, neo-Keynesians, institutionalists and, to a lesser extent, evolutionary economists, typically argue that the way mainstream economic theory deals with decisions under uncertainty is flawed. Their contention is that the modeling of uncertainty as a well-defined subjective probability distribution held by individual agents over a complete and exhaustive list of possible outcomes cannot take into account 'genuine' uncertainty. Sometimes it is also suggested that any formalized attempt to deal with genuine uncertainty is bound to be unsatisfying.

This paper centers mainly on the Austrian characterization of the mainstream representation of decision making. Take, for instance, this excerpt from the entry 'Risk and uncertainty' in the *Elgar Companion to Austrian Economics*. 'In the end, what is distinctive about Austrian and related understandings of risk and uncertainty is an emphasis on what we will call *structural uncertainty*: that is, a lack of complete knowledge on the part of economic agent about the very structure of the economic problem that agent faces'. The neoclassical focus, on the other hand, is on '*parametric uncertainty*', which presupposes 'certainty about the structure of the world'. But, 'as ordinary language suggests, …, one can take uncertainty to mean that one does not in fact know with certainty a listing of all possible states of the world'. This kind of uncertainty is 'not captured in neoclassical modeling and not well-suited to the probability calculus' (Langlois 1994: 118-120). Though with different emphasis, this characterization is shared almost unanimously in the Austrian circles, under various headings such as 'partial ignorance', 'Knightian uncertainty', 'radical uncertainty' and so forth. In this paper, following Vaughn (1994), we will identify the mainstream and the Austrian view concerning individual decision making under the heading of 'rational' versus 'radical' ignorance.

Our main goal in this paper is to show that, though it is surely true that the above mentioned is a main point of distinction of Austrian subjectivism from the standard neoclassical approach,[1] this characterization may be confounding on two different grounds: first, the attempts on the neoclassical side to deal with uncertainty are much more elaborated than conceded, and, second, the conclusion that probability calculus is basically unable to

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1 It is of course difficult to find a largely agreed criterion to classify which contributions can be labeled as part of the mainstream and which can not. For instance, Williamson (1985) is not a typical neoclassical representative. But his contribution draws on opportunistic behavior at least as much as standard neoclassical works. We take as the crucial point the fact of whether or not we are dealing with individuals who are utility maximizers. From this point of view, all the studies in decision theory we will be mentioning can be considered mainstream.

deal with radical ignorance (i.e., 'structural' uncertainty) is not proven. We do not yet seek to provide a general assessment of alternative approaches to decisions under uncertainty (for which see Kelsey and Quiggin 1992 and Vercelli 1999). We only intend to discuss the main analytical point related to the claimed impossibility by individual agents to know 'the structure of the world'. And this point is mainly Shackle's (1949 and 1961) point. Indeed, Shackle's viewpoint on probability theory as applied to decision making is mostly the one referred above. In fact, it is Shackle's contention that 'the very construction of the probability calculus relies on certain knowledge of the structure of the world, whereas in reality agents do not have such knowledge and, in particular, are not capable of enumerating all possible contingencies or "states of the world"' (Langlois 1994: 121). We shall argue that a closer examination of the recent developments in decision theory shows that the crucial role played by uncertainty in economic phenomena may find adequate formal treatment by means of a refinement of the standard instruments of probability calculus.

The paper develops into three sections. The next section deals with the distinction between rational and radical ignorance from an Austrian viewpoint. Instead of providing an assessment by our own, we find it convenient to use Karen Vaughn's (1994) assessment of the Austrian tradition as a reference point for analysis. The need for an individualistically based notion of order in the Austrian tradition is also considered. The section after next provides a specific analysis of the main analytical point in question: is there a formal representation for decision making which is apt to deal with a situation in which the listing of all possible states of the world is not known with certainty? The answer we provide, by means of an overview of some recent advances in decision theory (notably Ellsberg 1961, Schmeidler 1989, Dow and Werlang 1992, Gilboa and Schmeidler 1994), is that there is, and we propose to consider as a notable example the formal representation given by the approach of non-additive probabilities. The final section tries to argue why economists working in the Austrian tradition should take these developments into account.

Austrian subjectivism and the mainstream attitude towards individual decision making

The fact that the modeling of individual behavior is crucial to the whole of Austrian tradition is well illustrated by Karen Vaughn's assessment of the recent developments of the Austrian School. In her attempt to summarize what aspects can be considered as 'hard core' Austrian, Vaughn points to a widely agreed opinion on the assumption of perfect knowledge used in

neo-classical economics as heading the list of commonly shared tenets among Austrian scholars:

> Austrians agree with neoclassical economics that human beings attempt to act rationally to achieve their purposes. However, because human action always takes place in time and always under conditions of partial ignorance about the present and total ignorance about the future, a theory of market processes can be neither static in nature nor based on the assumption of perfect knowledge. Nor is *rational ignorance* a promising assumption for Austrians who deny that all the relevant future states of the world are listable by the choosing agent
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> (Vaughn 1994: 163, emphasis added)

This is why, Vaughn stresses, Austrian economics 'cannot usefully be considered merely a variation on the economics of rationality and constrained maximization' (Vaughn 1994: 162). This statement accounts for her position in the reconstruction of the diverging paths in modern Austrian economics, which is critical of Kirzner's ideas and supportive of Lachmann's. In order to clarify Vaughn's point, it is first worth dealing with her view of these two authors.

Vaughn argues that Kirzner has been successful in providing an analysis of the process through which competitive markets may reach equilibrium. His notion of the alert entrepreneur is a definite step forward in the appreciation of the role of those economic agents who 'notice opportunities that others miss and act upon that knowledge to bring markets closer to equilibrium' (Vaughn 1994: 165). But when real time and genuine uncertainty are taken into account there is no longer any reason to argue that each entrepreneur is 'correct' in his action, and thus no reason for expecting their joint actions to be equilibrating, as Kirzner assumes. This is why any attempt to formalize the entrepreneurial behavior as a problem of constrained maximization under uncertainty is bound to be unproductive. Genuine uncertainty must imply that the entrepreneur cannot anticipate all possible future consequences of his action. Therefore equilibrium cannot be considered an *ex ante* reference point for analysis. Lachmann's contribution, on the other hand, takes stock of the traditional Austrian emphasis on heterogeneous and incomplete knowledge and comes to the conclusion that only those descriptions of economic activities which consider endogenous and unpredictable change are apt to understand agents' behavior. The market process driven by individuals whose acting is 'undetermined creative choice' (Vaughn 1994: 152) is necessarily an open-ended process. As a result, not just the possibility of anticipating it *ant*e, but the very notion of equilibrium is called into question.

The main point of disagreement between the two alternative views of Kirzner and Lachmann can be better understood once the explanation of what the achievable aggregate outcomes are is also taken into account. They both recognize that the question is not so much that of incorrectly perceived opportunities by individual agents — which is usually dealt with in neoclassical economics — as that of unperceived opportunities. Still, Kirzner admits that equilibrating actions can be consistently defined, thus sticking implicitly to a (perhaps generalized) constrained optimization approach in the Robbinsian tradition. As Vaughn contends, 'he has improved upon a model of market behavior that still fails to capture the central problem of human action' (Vaughn 1994: 150). In fact, 'he rejects the notion that entrepreneurs create anything *ex nihil*o, instead arguing that by discovering opportunities already 'there' to be discovered, they are introducing genuine novelty into the system' (Vaughn 1994: 148). Thus it would seem that, from Vaughn's viewpoint, 'genuine novelty' and unperceived opportunities are distinct aspects of the environment relevant for the act of decision making.

In contrast, Lachmann thinks that, if equilibrium is no longer a useful tool, then the notion of equilibrating action is unintelligible. Therefore he argues for an entirely different approach. Following Shackle, Lachmann maintains that it is the undetermined nature of the future that explains why the consequences of creative choice are unpredictable. 'Genuine novelty' rests in the fact that 'no two minds are alike', so that neither individual choices nor their outcome can be fully predicted. As a consequence, economic theory must draw on the notion of a plan 'to make world intelligible in terms of human action'. But the passage of time accounts for the fact that 'revision of plans is the norm rather than the exception' (Vaughn 1994: 153, 154), thus rendering coordination almost unachievable as a state of the economy. Lachmann's suggested solution is then to be sought, Vaughn contends, in the study of those institutions which can favor order even in the face of uncoordinated patterns of behavior.

But, as regards the possibility of having a formal theory of economic decisions, we are only left with a series of negative statements. In Vaughn's words 'Lachmann, in an attempt to take radical subjectivism and real time seriously in his interpretation of economic action, tries to devise an alternative to equilibrium theorizing but fails to produce the kind of overall theoretical structure that would seriously challenge the neoclassical hegemony' (Vaughn 1994: 161). In particular, one might add, Lachmann does not provide any description of the characteristics of the domain encompassing not only unperceived but, what is more relevant here, also genuinely novel opportunities. Lachmann (1976), as most Austrians do, refers to

Shackle's approach to decision theory. A closer examination of the reference to Shackle's viewpoint is postponed to the section after next.

If one looks at the theory of decision adopted by mainstream economic theory for the last forty years, Vaughn's reconstruction of the Austrian diversity appears to be plain. The theory of economic decisions has been based — starting from Savage's definition of states of nature — on Bayesian decision theory, which requires that the possible events must be 'listable' and that their (objective or subjectively retained) probability of realization add up to unity. Indeed, the basic assumption of the standard approach to decision theory under uncertainty is that economic agents know with certainty the domain of their uncertainty. This is of course not a theoretically appealing assumption if one is interested, as the Austrians are, in 'themes such as the importance of dynamic growth and development, the generation and function of knowledge in economic action, the uncertainties associated with processes in time and the pivotal importance of diversity and heterogeneity in economic life' (Vaughn 1994: 162).

But, given that perfect knowledge is obviously not a common assumption in most modern economic theory, the main question that the comparative analysis of the 'hard core' Austrian and neoclassical different positions leaves open is the following: what does *rational ignorance* really mean?

In the first instance, it is worth noting that in recent years some perceptive mainstream economic theorists have shown that they share with Austrians the same discomfort with respect to the difficulties in representing formally how the knowledge of individual agents changes — which is the upshot of Lachmann's insistence on the subjective nature of knowledge. More and more of them have increasingly acknowledged these difficulties. This is apparent, first of all, in the evolution of the notion of equilibrium toward a more dynamic conception, as in the works of Hahn (1973 and 1989) and Fisher (1983). For instance, in their examination of the models provided by Hahn and Fisher, Currie and Steedman (1990: 215) find it 'striking that much recent work has more in common ... with Lachmann's conception of market processes than it does with the Arrow-Debreu economy'. Currie and Steedman's opinion is relevant with respect to our argument because their analysis of those economists who, in the history of economics, have dealt with the behavior of economies over time highlights exactly the importance of Shackle's and Lachmann's contributions.

Moreover, and this is more important with respect to the Austrian themes we are dealing with, there is now an increasing number of attempts to explicitly deal with the question from a choice theoretic perspective. To take a notable example, Williamson's (1985)

contention that many of the forms of contractual arrangements one can observe in markets and organizations are to be attributed to the need to adapt to contingencies which cannot be anticipated at the date of the signature of the contract has originated an entire literature on incomplete contracts. In the literature on incomplete contracts the argument for signing incomplete contract conceives fully rational agents who decide not to spend time in describing states which in principle can be described (for instance, because they are not easily observed from outside by a judge, as suggested in Hart and Holmstrom 1987). In this instance agents may form beliefs which can be represented as probabilities over the set of unexplored states. In order to provide a choice theoretic foundation to Williamson's point, Kreps (1992) provides a model of choice in which the individual agent is aware, at the outset, that unforeseen contingencies may arise — in other words, the individual agent might not have been able to imagine at an earlier date an event which he now has to face up to. This can accommodate for the idea of different degrees of flexibility preserved by agents for future decisions about possibly new events (Kreps 1992, 259-61). One can also refer to the related, and probably more powerful, notion of unawareness presented in Modica and Rustichini (1994), where the discussion involves both unforeseen and unforeseeable contingencies — in other words, the individual agent might have been unable not only to think of the event but even to understand it before its realization. A recent influential review on incomplete contracts by Tirole (1999: 772), though arguing in favor of a complete contract methodology, recognizes that 'recent developments on the relaxation of the Savage axioms for an individual decision maker trying to capture "unforeseen contingencies" are clearly welcome'.

It is also worth stressing that even if one points at the efforts of general equilibrium theorists, it can be shown that they are now involved in accounting for endogenous uncertainty and information asymmetries (Magill and Quinzii 1996). The standard framework is one of missing markets and impossibility of complete insurance against future events. Indeterminacy, that is multiplicity of equilibria, is regarded as the norm and Pareto-constrained efficiency of equilibria is not guaranteed. This kind of approach hints at a departure from traditional choice theory which is not in principle limited to exogenous uncertainty, as indicated by Hahn's conjecture (1995) about the possibility of introducing endogenous uncertainty into equilibrium theory via the notion of unawareness (see also Arrow and Hahn 1999).

The aspects of decision theory just mentioned do not represent an isolated contention by certain leading authors. The astonishing increase of the number of studies concerned with informational asymmetries, incomplete contracts, non-additive probability theory and so on

does not simply entail a major extension of formal exercises in constrained maximization. These studies indicate which is the attitude of modern economic theory toward the question of rational ignorance. The traditional formulation of the problem of decision under uncertainty is still dominant, above all because of the central role assigned to Bayesian individuals maximizing expected utility. But an increasing number of papers in leading mainstream journal are devoted to the study of alternative ways of formalizing uncertainty exactly because it has been recognized that radical ignorance is something different from Bayesian rational ignorance. In Hamouda and Rowley's (1997: *xx*) words – that is, once more from a supportive side of Austrian views – 'while many textbooks retain and stress the notions of probability as established by the beginning of the 1970s, two decades of active innovation with vague and imprecise alternatives has undermined earlier myopia and complacency, widened the conventional structure of policy analyses involving uncertainty, produced some means of translating common forms of imprecision into useful ingredients for modeling frameworks, and thus generated a less hostile audience for unconventional views of uncertainty and their application to real phenomena'.

To sum up on this question of rational ignorance, our point is that the comparison between neoclassical theory and alternative paradigms, such as the Austrian, should take into account the multiple aspects of neoclassical theory. Since the question of how to formalize decision making under uncertainty is central in much of modern economic theory, and the suggested solutions cannot be simply considered variations in constrained optimization, we find it unhelpful to compare the Austrian insights on knowledge and time with the traditional corpus of neoclassical theory. We delayed until the next section the issue of which approach we think is best suited for dealing with these themes.[2]

Before moving on it might be worth considering an important outcome of Vaughn's assessment which we find relevant for the debate within Austrian scholars about how to develop a modern Austrian view. We have seen that while she considers Kirzner's analysis too closely linked to the mainstream, Vaughn finds it difficult to clarify the Lachmannian alternative. Lachmann's attitude toward what other Austrian scholars have characterized as 'theoretical nihilism' seems to leave economic theory without a clear path to follow, at least as regards the study of individual behavior. Vaughn's viewpoint is that the solution to this

2 It must be stressed from the outset that these studies (and many others, for which see Kelsey and Quiggin 1992) do not constitute a coherent entity. For instance, Kreps's formalization holds only if the 'sure-thing principle' is assumed, while studies in non-additive probability theory are based precisely on its denial, which emerges from the Ellsberg paradox (for example see Machina 1987, and Camerer and Weber 1992). But on this point the reader is asked to wait until next section.

problem can be found in an elaboration of the notion of order which takes the role of institutions into due account. But the attitude that denies the possibility of any solution at the individual level is unconvincing.[3]

Vaughn's view is that Austrians', an particularly Hayek's, insistence on the heterogeneous and dispersed nature of market knowledge not only implies a vision of the market order as a discovery procedure, 'a means of inducing individuals to learn more about the opportunities available to them and to create new products and new methods of production'. It also allows an 'evolutionary theory of social institutions wherein those that survived only did so because they better helped individuals within a society to achieve their goals'. It is indisputable, Vaughn concludes, 'that [Hayek's] theory of social evolution helped to point Austrian economists toward the study of economic institutions and evolutionary orders in a systematic way' (Vaughn 1994: 126-7). The Austrian alternative to conventional equilibrium theorizing is thus to be found in the development of an evolutionary theory of institutions. The implicit assumption in Vaughn's reading of Hayek and the subsequent evolution of the Austrian paradigm is that a specific analysis of individual behavior no longer matters once the methodological implications of Hayek's work are correctly drawn. The Hayekian notion of spontaneous order is not to be interpreted simply as a fundamental shift in thinking about the meaning of the type of co-ordination that is conceptualized by general equilibrium (as in Moss 1994). Neither can it be interpreted as a qualitative equilibrium construct within which formal economic theory can help in clarifying the phases of plan co-ordination. On the contrary, it is a definite step toward an understanding of economics largely as 'a study of economic institutions within a non-equilibrium context' (Vaughn 1994: 127). This is why Kirzner's approach to individual behavior is regarded merely as a variation in

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3 Moreover, in our view, it is fruitless in the comparison between Austrian insights and the mainstream, because it makes it difficult to understand whether the mainstream has actually understood the Austrian message. To take an example, in the Austrian literature the influences of Hayek's work on the economics of information are often noted (among others, see Kirzner 1984 and Thomsen 1992). But the unanimous conclusion by the Austrians is that Hayek's profound insights have been misunderstood and not properly dealt with. It is worth noting that non-Austrian theorists show an opposite attitude on this historiographic matter. Hahn (1990) has contended that a typical Hayekian theme such as that prices may reflect the different expectations of agents and thus reveal information has been carried well beyond Hayek's vague remarks, and 'fully absorbed' in neoclassical economics, only by virtue of the literature on revelation of information prompted by Radner (1979) and Grossman (1989). Milgrom and Roberts (1992, Ch. 4) consider Hayek's notion of personal knowledge central but insufficient on its own for the comparative analysis of organizations and the market. Similar viewpoints can be found in Arrow (1994), with respect to the notion of personal knowledge as compared to that of technological knowledge, and in Bowles and Gintis (1993) and Stiglitz (1994), with respect to those functions performed by the market which are not simply allocative. On this point see also Zappia 1998. One of the two authors of this paper has argued elsewhere that an attitude different from that shown by most of the authors writing in the neo-Austrian tradition with respect to the relationships between Hayek's insights and modern attempts to deal with asymmetries of information may be more fruitful (Zappia 1997)

constrained optimization. As for Lachmann, his inability to give analytic content to his insights on the inherently continuous revision of individuals' plans is deemed unimportant in comparison to the alternatives to conventional equilibrium theorizing suggested in his work. One might even argue that Vaughn's assessment of Lachmann's role in the development of Austrian thought points to the 'beneficial' influences of Lachmann's belief that no formal theory of individual decision making can be arrived at.

Vaughn's proposal then is to follow the implications of her reading of Hayek's abandonment of general equilibrium analysis, that is to investigate a different notion of order. But Hayek's notion of spontaneous order in itself cannot accomplish this task. For Vaughn's contention about the impossibility of giving formal support to the analysis of individual behavior when new knowledge is prompted by the passage of time implies that the market tendency toward a spontaneous order is not guaranteed. It is at this point that Vaughn's discussion of those institutions that 'permit the use of new knowledge in human action' (Vaughn 1994: 174) is not very satisfying. She only refers to the tradition of those economists, notably Nelson and Winter, who have attempted to adapt evolutionary reasoning to economic processes, and to the similarities between certain features of evolutionary theory and the Austrian viewpoint, as represented by Witt (1992) and Horwitz (1992). She also concedes, 'there is much work to be done' (Vaughn 1994: 175). Indeed, the need for the Austrian research program to abandon the equilibrium metaphor and to elaborate an evolutionary notion of social order is supported only by a few suggestions for future research. However Vaughn does not even discuss the difficulties to reconcile methodological individualism with group selection processes on which the evolutionary approach hinges.

The future relevance of Austrian economics might probably depend on the viability of Vaughn's suggestions, but Vaughn does not discuss why the analysis of economic institutions cannot be based on the study of individual behavior, as it is traditional in the Austrian approach. She argues that 'people carry out their projects and plans within a variety of social institutions, all of which have both tacit and explicit rules of behavior. ... . Indeed, an agreement between two people to engage in a recurrent pattern of behavior *vis á vis* each other is also a form of 'institution' or typical behavior' (Vaughn 1994: 171). But here Vaughn neglects to refer to the fact that a leading interpretation of the recent developments in the economics of information is that if opportunistic behavior is properly taken into account, then the typical contract between two asymmetrically informed agents can be interpreted as the outcome of tacit rules of behavior (for a summary, see Bowles and Gintis 1993). A more thorough inspection of the market as an institution reveals that many aspects of economic

activity such as repeated interaction for exchange purposes do not necessarily involve the emergence of organizations, but can be explained instead as the emergence of conventional behavior among distinct market participants, and that this can be interpreted as 'a form of "institution"'. This view is consistent with the Austrian view of the market as represented by Hayek. Hayek's conception of the superiority of the market over alternative organizational settings is not exclusively linked to the impersonal working of the price system and its efficiency in diffusing existing knowledge and creating the incentives for discovering new knowledge. It also emphasizes the role of those forces of competition, such as imitative behavior, rules and traditions, which were excluded by the Walrasian interpretation of competition. The view that the exchange of information which is dispersed throughout the system is achieved through a process which is more complex that the Walrasian process of impersonal allocation through prices is not only compatible with Austrian thought, but has also been strongly supported by Hayek (1948 and 1968).[4]

Here a paradox seems to emerge: following Vaughn's reconstruction of Austrian thought, it might be argued (as in Bowles and Gintis 1993) that new developments of what Vaughn considers neoclassical theory have done more than the Austrians for providing an individualistically-based explanation of those elements — such as habits and customary business procedures — which characterize economic institutions. But this is of course untrue, as Vaughn herself stresses in her reassessment of Hayek's theory of knowledge and the related efforts by O'Driscoll and Rizzo 1995) to develop an economics of time and ignorance in which 'the existence of private and tacit knowledge implies that nonprice signal can contain important market information' (Vaughn 1994: 136).

Radical ignorance: the non-additive approach to probability

We have argued in the previous section that, though with different emphasis, Austrian scholars consider the mainstream approach to decision under uncertainty flawed. This section

4 As one of the two authors has argued elsewhere (Zappia 1999), a superficial denial of the relevance of many recent microeconomic developments to understanding the market as an institution is inconsistent with Hayek's insights into the matter. But see also the 'Austrian rationale' for the existence of organizations provided by Minkler 1993.

examines the main analytical point at issue, that is the assumption made by the mainstream that the agents knows with certainty the domain of his uncertainty.

The traditional approach to decision under uncertainty is structured as follows. Two are the fundamental assumptions. First, there is available to the individual a complete list of possible future states of the world – which, in an interpersonal context, is common knowledge to all individuals – and the individual is endowed with subjective beliefs over the state space that are represented by a well-defined (additive) probability function. This is due mostly to Savage's (1954) expected utility theory, which made it possible to apply all rules of probability theory to a belief representation. Indeed, Savage's expected utility theory is typically referred to as providing a subjective theory of probability because the probability measure underlies choice behavior. More precisely, it is derived from axioms on the preference ordering of uncertain prospects, that is acts defined on the state space, and serves as a component in the representation of that preference. As a result, in an uncertain context, individuals are supposed to be able to undertake expected-cost/expected-benefit analysis in information gathering and reach an informational optimum. This is why it is correct to say that they are *rationally* ignorant (or 'probabilistically sophisticated', as in Machina and Schmeidler 1992). Second, the processing of information consists of the Bayesian updating of the individual's belief (prior probability distribution) when he/she receives a signal on the realization of the state. This is the outgrowth of the implicit assumption that the individuals are rational in the strong sense that they can deduce every logical proposition that can be deduced with respect to the axioms of the theory, which means that the requirements of means-rationality are satisfied (Hamlin 1986). This second assumption is of course highly questionable after Simon (1982), and is abandoned, to a certain extent, in boundedly rational and evolutionary models (see in particular Nelson and Winter 1982), but we shall not explicitly discuss computational and cognitive problems in our analysis and concentrate on the first assumption.

Let us start from a slightly more specific description of how the individual problem is dealt with under the first assumption. Decision theory under uncertainty describes how an individual makes and/or should make a decision among a set of alternatives, when the consequences of each action are tied to uncertain events whose probabilities are subjective. The individual formalizes the problem setting alternatives (technically acts), states of the world and consequences. The individual acts on the basis of a well-defined utility function representing his/her preferences that involve an evaluation of consequences and their likelihood. The rational decision-maker's goal is to maximize his/her expected utility in the

case in which probabilities are objective (von Neumann and Morgenstern 1944) or subjective (Savage 1954). Both these theories and their mixed (horse-race/roulette wheel) version (Anscombe and Aumann 1963) weight consequences by a unique probability measure on the set of states of the world and in such a way they induce the linearity of the preference functional. As a consequence, the expected utility can be represented as the mathematical expectation of a real function on the set of consequences with respect to a unique probability distribution. Linearity in the probabilities is a direct consequence of two very similar axioms, that is the 'independence axiom', in von Neumann-Morgenstern's theory, and the 'sure-thing principle', in Savage's theory. The independence axiom states that given two alternatives (lotteries in technical language), each of them composed of an action and a certain common act, preferences between them should be independent of any common consequence with identical probability (common act). The sure-thing principle assumes that the decision-maker ignores states in which actions yield the same consequences when choosing between the actions.

Let us now retain the Bayesian assumption that, in principle, individuals are able to formulate a unique subjective probability distribution in order to deal with any kind of uncertain situation, but question the reliability of this distribution when there is awareness that a possible future event can happen which is not in the list, or when the decision to be taken is conditioned to a non-repetitive event. As it is well-known, in the development of modern decision theory the study of the possible unreliability of the subjective probability distribution was not, so to say, in the agenda for future research. In fact the issue came out as a result of experimental evidence which revealed systematic violations of Savage's 'sure-thing principle' that are inconsistent with the hypothesis of expected utility maximization. The most discussed of such violations are the Allais Paradox (Allais 1953) and the Ellsberg Paradox (Ellsberg 1961). The Allais Paradox is a seminal counterexample to the validity of the expected utility theory but it shows a puzzle built on elements of certainty, small probability difference, high versus low stakes, common consequences etc. On the contrary, the challenge posed to the expected utility theory by the Ellsberg Paradox can be considered the most crucial, because it focuses on the belief side of the decision problem and involves considerations about ambiguity and confidence.

The following experiment is due to Ellsberg (1961). An individual faces an urn which contains 30 red balls and 60 balls in some combination of black and yellow: there is no information whatsoever about the number of black and yellow balls in the urn (unknown proportion). A ball will be drawn from the urn. There are two pairs of acts, X and Y, and X'

and Y'. Acts have consequences of 1 or 0 as follows: choosing X one gets 1 if red and 0 if black or yellow, choosing Y one gets 0 if red or yellow and 1 if black, choosing X' one gets 1 if red or yellow and 0 if not, choosing Y' one gets 0 if red and 1 if black or yellow. Ellsberg reported that most of the people asked choose X instead of Y and Y' instead of X' thus revealing a remarkable preference for betting on known probabilities of winning. That is, it appeared that confidence in estimates of subjective probabilities is taken into account by individuals when making choices. These decisions are inconsistent with Savage's sure thing principle. In fact both pairs of acts only differ in consequences when the yellow state occurs, and these consequences are the same for X and Y (the individual gets 0) and for X' and Y' (the individual gets 1).

Moreover, the beliefs of the individual exhibiting such preferences cannot be represented by an additive probability distribution. Suppose p(r), p(b) and p(y) are the subjective probabilities of each possible draw. Setting U(0)=0, Savage's subjective expected utility implies that X is to be preferred to Y if and only if $p(r)U(1) > p(b)U(1)$ or $p(r)>p(b)$. Likewise Y' is preferred to X' if and only if $p(b\cup y)>p(r\cup y)$. This contradicts the assumption that probabilities are additive: in fact, given $p(b\cap y)=0$, if $p(b\cup y)=p(b)+p(y)$ then to prefer Y' to X' implies $p(b)>p(r)$ which conflicts with what is implied by preferring X to Y, that is, $p(b)<p(r)$.

As a result, these preferences contradict the expected utility theory and every other theory of rational behavior under uncertainty that assumes a unique additive probability measure underlying choices. In Ellsberg (1961: 654) words, 'it is impossible, on the basis of such choices, to infer even qualitative probabilities for events in question…to find probability numbers in terms of which these choice could be described - even roughly or approximately – as maximizing the mathematical expectation of utility'.

Violations of both 'complete ordering of actions' and the 'sure-thing principle' pointed out by Ellsberg (1961) in both two-urns and two-color balls and one urn three-color balls hypothetical experiments, have been confirmed through a lot of experiments replicated in recent years (Camerer and Weber 1992). This suggests that most agents prefer making unambiguous choices than ambiguous ones. Individual choices are not affected by 'the relative desirability of the possible payoffs and the relative likelihood of the events affecting them, but …the nature of one's information concerning the relative likelihood of events. What is at issue might be called the *ambiguity* of this information, a quality depending on the

amount, type, reliability and unanimity of information, and giving rise to one's degree of confidence in an estimate of relative likelihood' (Ellsberg 1961: 657).

Let us now consider a crucial aspect of Savage's decision theory that has been often forgotten in application of his theory, indeed the distinction between the *grand world* and the *small world*. The grand world is the complete list of states (objects or anything at all) about which an individual is concerned. The small world is a construction derived from a partition of the grand world into subsets or small world states, which are subsets or events of the grand world. As a result, 'the small world is determined not only by the definition of a state, but also by the definition of small world consequences [since] a small world consequence is a grand world act' (Savage 1954: 85). Savage affirms that an individual has to confine his/her attention to relatively simple situation in almost all his/her decision, that is, for practical necessity, the individual is concerned with a small world, 'derived by neglecting some distinction between states and not by ignoring some states' (Savage 1954: 9) of the grand world. Considering a small world as crucial for his/her decision, the individual is describing states of the world and consequences at a limited level of detail. It is worth to note that the individual can always consider a more refined and detailed small world until he/she arrives to the grand world that takes everything into account. The bug of Savage theory is in the fact that a small world "is completely satisfactory only if it is actually a *microcosm*, that is only if it leads to a probability measure and a utility well articulated with those of the grand world' (Savage 1954: 88). If there is no question about utility, there is no certainty that the probability of an event in the small world equals the probability on the corresponding collection of subsets in the grand world. If the probabilities are different for the two levels of refinement, probabilities attached in the small world are right only if they are the same as the ones calculated from the grand world. The individual has to be able to work with the grand world, since the condition that assures equality in probability 'seems incapable of verification without taking the grand world much too seriously' (Savage 1954: 90). There is a tricky question: the individual only works in a practical device called the small world, but he/she is able to exhaustively enumerate all the possibilities in advance and explore in detail all the consequences, indeed he/she has a *Divine Knowledge*. As Savage is careful to claim (1954: 16, 84), the subjective expected theory is to be apply only to small worlds[5] when 'all the possibilities can be exhaustively enumerate in advance, and all the implication of all

[5] It is worth to note that Savage refusal of Allais paradox is mainly based on the distinction between the grand and the small world (Savage 1954: 101).

possibilities explored in detail so that they can be neatly labeled and placed in their proper pigeonholes' (Binmore 1990). In situations in which outcomes and states are not clearly given in the description of the problem, it is clear neither what the normative implications of Savage's sure thing principle are nor why Savage's expected utility approach could inform actual behavior.

Even if uncertainty encompasses the intuitive concepts of ambiguity and vagueness, it is possible to provide a more precise notion of uncertainty as compared to risk by dealing with the description of the world. Consider a decision problem in which the states of the world included in the model do not exhaust the actual ones. A description of the world might be considered as a misspecified model whenever certain states are not explicitly included in the model. When an individual agent does not know how may states are omitted, one can represent his/her beliefs by either a non-necessarily-additive measure or a set of additive probability distributions on the set of events. Gilboa and Schmeidler (1994) show that a decision model with non-additive measure on the state space may be embedded in a decision model with an additive measure (probability) in which the enlarged state space includes all the possible missing states. As a result, it is possible to relax the non-additivity of a measure at the expense of the dimension of the decision model.

In the representation of uncertainty by a non-additive measure on the space state the relationship between the epistemic status of the individual (awareness of incomplete knowledge and reliability of likelihood assessments) and his/her choice is implicitly assumed. Mukerji (1997) clarifies this relationship on the basis of a two-tiered state space model that embedded 'a space on which the individual assigns primitive beliefs and a space of payoff relevant states, i.e. states on which the available acts are directly defined' (Mukerji 1997: 25). The two-tiered state space modeled by Mukerji happens to be mathematically isomorphic to the enlarged space of Gilboa and Schmeidler. The individual assigns his/her beliefs (priors) on his/her perceived simpler state space (primitive) and then he/she 'infers beliefs about the events to which the outcomes of acts are directly related' (Mukerji 1997: 25), that is called derivative world. We think that it is straightforward interpret the primitive and derivative world as the small and grand world r, respectively. The primitive state space (the small world henceforth) is a set of objects on which the individual has direct experience, clear intuition and empirical knowledge and belief assessments on this device state space express this confidence. Likelihood assessments on the derivative world (the grand world henceforth) are deduced by an *implication mapping* that embodies the individual knowledge of the association between the two world. As a result 'the decision-maker's knowledge about the

likelihood of an event in the derivative frame is given by the sum of the beliefs assigned to those elements of the primitive frame whose implications are sub-events of the event in question….however depending on the epistemic condition informing the agent's situation, the beliefs on the derivative frame may have a non-additive representation' (Mukerji 1997: 33). In fact, if the individual transfers a likelihood assigned to the small world to an event in the grand world, that is he/she is unable to distribute the beliefs across the elements of the grand world world, then 'non-additivity is merely an expression of the bounds on the decision-maker's understanding of the possibilities of the world and of his/her awareness of those bounds' (Mukerji 1997: 33). It is straightforward to assume that an individual behaves as if he/she has a set of priors or a non-additive measure rather than a well-defined probability if his/her perception of grand world is fuzzy., incomplete or vague.[6]

Summing up, we shall say that a decision-maker faces Knightian, radical uncertainty if he/she has a misspecified description of the states of the world, he/she is unable to assign a reliable probability distribution to states of the world because they are ambiguous, he/she has ignorance of the world in which he/she has to act and attaches an interval of probabilities to each event.

Two main non-expected utility theories have been proposed to encompass uncertainty attitude (versus risk attitude) and the expected utility maximization. Gilboa (1987) and Schmeidler (1989) axiomatize a generalization of expected utility, which provides a derivation of both utilities and non-necessarily-additive probability by the Choquet integral (Choquet 1954). Gilboa and Schmeidler (1989) extend the classical expected utility representing preferences by means of a utility function and a set of additive probabilities, instead of a unique additive one on the set of events. It is worth stressing that the two approaches coincide with respect to the issue we are concerned here.

Let $\Omega=\{w_1,...,w_n\}$ be a non empty set of states of the world and let $S=2^{\Omega}$ be the set of all events. A function $\mu:S\rightarrow R_+$ is a non-necessarily-additive probability measure or a capacity if it assigns a value 0 to the impossible event $\varnothing$ and value 1 to the universal event Ω, i.e. the measure is normalized, and for all $s_1,s_2\in S$ such that $s_1\supset s_2$, $\mu(s_1)\geq\mu(s_2)$, i.e. the measure is monotone.

A capacity is convex (concave) if for all $s_1,s_2\in S$ such that $s_1\cup s_2$, $s_1\cap s_2\in S$, $\mu(s_1\cup s_2)+\mu(s_1\cap s_2)\geq(\leq)\mu(s_1)+\mu(s_2)$ and μ is super-additive (sub-additive) if

6 In this way it is possible to make explicit the Savage's distinction between sure and unsure probability relations (Savage 1954: 58).

$\mu(s_1 \cup s_2) \geq (\leq) \mu(s_1) + \mu(s_2)$ for all $s_1, s_2 \in S$ such that $s_1 \cup s_2 \in S$, $s_1 \cap s_2 = \varnothing$. Since μ is a non-additive measure, the integration of a real-valued function $f: \Omega \rightarrow R$ with respect to μ is impossible in the Lebesgue sense and the proper integral for a capacity is the Choquet integral. The Choquet integral with respect to a capacity is a generalization of the Lebesgue integral and it requires that states of the world have been ranked from the most to the least favorable ones, or vice versa, with respect to their consequences.[7] As a result, the Choquet integral is a generalization of mathematical expectation with respect to a capacity.

Following this approach, it can be said that the decision-maker expresses Knightian uncertainty aversion (respectively love) if he/she assigns larger probabilities to states when they are unfavorable (respectively favorable), than when they are favorable (respectively unfavorable), that is if his/her non-additive measure is convex (respectively concave) (Dow and Werlang 1992). Hence, the convexity of the capacity captures the decision-maker's Knightian uncertainty aversion and encompasses the conservative statement that the decision-maker acts 'as though the worst were somewhat more likely than his best estimates of likelihood would indicate [and] he distorted his best estimates of likelihood, in the direction of increased emphasis on the less favorable outcomes and to a degree depending on his best estimate' (Ellsberg 1961: 661).

Alternatively, Knightian uncertainty might be represented by a set of possible priors instead of a unique one on the underlying state space, that is the individual does know enough about the problem to rule out a number of possible distributions. In this case the agent has multiple additive probability measures P on $\Omega = \{w_1, ..., w_n\}$ and his/her preferences are compatible with either the maximin or the maximax expected utility decision rule.[8] In fact, if the agent is Knightian uncertainty averse, he/she maximizes the minimal expected utility with respect to each probability in the prior set, thus $\int f \, dP = \min \int f \, dP$ such that $p \in P$. On the contrary, if the agent is Knightian uncertainty loving, he/she maximizes the maximal expected utility with respect to the set P, thus $\int f \, dP = \min \int f \, dP$ such that $p \in P$. Gilboa and Schmeidler (1989) and Chateauneuf (1991) have proved that when an arbitrary (closed and convex) set of possible priors P is given and one defines either a non-additive probability measure υ (convex) or ν (concave) on Ω, such that all additive probabilities measures in P

[7] The Choquet integral of f with respect to μ is

$$\int f d\mu = \int_0^\infty \mu(\{w | f(w) \geq t\}) dt + \int_{-\infty}^0 [\mu(\{w | f(w) \geq t\}) - \mu(\Omega)] dt$$

majorize υ or minorize ν, the non-additive expected utility theory respectively coincides with the maximin or the maximax decision rule. The non-additive expected utility with respect to a convex (respectively concave) capacity and the maximin (respectively maximax) expected utility give the same solution if P is considered the core of υ (respectively ν), at least a proper subset of the core of υ (respectively ν), since by definition the core of υ (respectively ν) consists of all finitely additive probability measures that majorize υ (respectively minorize ν) event-wise.[9]

Let us take stock of our analysis so far. For both theoretical and empirical reasons economists working in decision theory have sought to generalize the expected utility model. At the basis of this development there is the distinction between risk and uncertainty usually attributed to Knight. Though this distinction is deemed to be unimportant for scholars working in a Bayesian perspective (for textbook evidence see Hirshleifer and Riley 1992), we have briefly referred to an axiomatic development which tries to incorporate such a distinction. The model discusses an individual maximizing expected utility with a non-additive probability by means of which it is admitted that the subjective probability that either of two mutually exclusive events will occur is not necessarily equal to the sum of their objective probabilities, so as to reflect the individual's attitude to uncertainty. What is more important, the axiomatization provides the basis for dealing with situations in which the uncertainty of the individual may concern the existence of a third (or others) event, to the possible occurrence of which no probability was attributed in the first instance.[10] If we come back to our distinction between the traditional assumption of rational ignorance and the advocated radical ignorance, this approach stands as a consistent attempt to give concrete, operational meaning to the second assumption. A number of important application have been proposed especially as regards financial markets (for instance Epstein and Wang 1994) and environmental problems concerning irreversibility (for instance Basili and Vercelli 1998).

Our main focus in this section has been to point out that a simple and straightforward modification of the axiomatic system of the traditional Bayesian approach to decision making

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8 The maximin (maximax) expected utility postulates that an agent with multiple priors looks at the least (most) value of expected utility for any act and chooses that act for which this least (most) value is greatest. See Arrow and Hurwicz (1972).

9 The details are developed in Basili 1999, Appendix I.

10 It is worth stressing that the representation of beliefs by real-valued set functions which do not necessarily satisfy additivity is not new: in particular 'belief functions' were introduced by Dempster (1967) and Shafer (1976). Though these theories were not directly related to decision under uncertainty it turned out that 'beliefs functions' are a special case of non-additive measures (or capacities) (Gilboa and Schmeidler 1994).

under uncertainty has allowed both to solve many of the emerging descriptive paradoxes and to face problems which are related to genuine uncertainty, such as the structural inability by the individual agent to know in advance the domain of his uncertainty which we labeled radical ignorance.[11]

Assessing Austrian subjectivism: some concluding remarks

Choice behavior such as that exhibited in the Ellsberg Paradox and related evidence have demonstrated that many decision-makers do not assign probabilities to all possible events. In situations where some events are 'ambiguous', decision-makers may not assign probabilities to those events, though the likelihood of 'unambiguous' events are represented in the standard probabilistic way. On the other hand, decision theorists recognize to an increasing extent that 'the fundamental assumption that the state space (either objective or subjective) is known to the individual … is problematic', and try to base a theory of decision making under uncertainty on the assumption that 'the individual knows the set of available actions and he knows that payoffs occur to each action in each period. But he has no further knowledge of the decision problem he is facing. In particular, states are not a part of his view of the world. He does not necessarily have knowledge of the objective or even some subjective state space' (Easley and Rustichini, 1999: 1158). As a result, the attempts to provide an axiomatic foundation for decision making in complex environment point exactly to the inadequate nature of the assumption about the structural knowledge of the environment by the individual.

The main problem which emerges is how to represent the individual agent's confidence in a probability assessment. This was the focus of drastic alternatives to probability models involving considerations about genuine ignorance, surprise and vagueness such as Shackle's (1949 and 1961). In Shackle's (1972: 15) view, the standard (Bayesian) meaning of probability 'stands for a *language for expressing judgements* as to the weight that the individual in choosing his conduct ought to give to each of a variety of rival hypothesis concerning the outcome of some one course of conduct … This language assumes, implicitly, that the hypotheses which have been enumerated, specified and presented for the assignment

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Likewise Zadeh's (1978) theory of fuzzy sets has been shown to be compatible to the non-additive probability approach (Wakker 1990).

11 For an assessment of the potential for conceptual generalization of theories of decision under uncertainty of the non-additive approach, see Vercelli 1999.

of weights are *the only relevant ones*. Thus the language of subjective probability is confined to the expression of a certain kind of meaning. And there are other meanings whose exclusion would be arbitrary and senseless.' As a result, Shackle's proposal on how to deal with uncertainty was to substitute some novel concepts such as potential surprise, epistemic probabilities, focus values and so on, for probability distributions. These concepts were intended to reflect both the mental processes and the non-repetitive, often irreversible, nature of actual economic decisions. Shackle emphasized the non-additivity of potential surprise and focused on the subjective, idiosyncratic nature of human judgements (for a concise assessment see Hamouda and Rowley 1996, Ch. 4).

But, we have tried to argue, the reliance of standard Bayesian theory on probabilistic judgements based on point-probability estimate which Shackle intended to contrast is no longer a justification for dispensing with probability calculus once the non-additive probability approach discussed above is considered. Shackle's (1961: 49-50) distinction between distributional uncertainty variables, that can be used if 'the list [of suggested answers to a question] is *complete without a residual hypothesis*', and non-distributional variables, that must be used when 'the list in order to attain formal completeness must be rounded off with a residual hypothesis' reflects an essentially non-additive characteristics of his theory. This was clear to Shackle from the very beginning of his effort. In a response to some critics of his first work on *Expectation in Economics*, he clarified that his system was non-additive because in order to describe 'mental states of uncertainty' what is need is 'a measure of acceptance, of a hypothesis proposed in answer to some question, which shall be independent of the degrees of acceptance simultaneously accorded to rival hypothesis', that is, 'a measure of acceptance by which the individual can give to new rival hypotheses, which did not at first occur to him, some degree, and even the highest degree, of acceptance without reducing the degrees of acceptance accorded to any of those already present in his mind (Shackle 1949-50: 70).

This is the main analytical point at the basis of Shackle's theory and the main analytical reference of Austrian economists dealing with the problem of how to represent decisions under genuine uncertainty. As we have recalled in our paper this is the crucial aspect upon which the distinction between the rational and the radical ignorance approaches has been drawn. We have tried to show that, contrary to Shackle's point, this distinction does not coincide with the distinction between the use of probability calculus and other forms of analysis.

As we have seen in the first section, the assessment of future Austrian developments which calls for a definitive shift in Austrian thought away from the search for equilibrium constructs and towards the analysis of those institutions which favor ordered outcomes of the market process implies an inevitable withdrawal from methodological individualism which is implicit in the endorsement of an evolutionary approach to economic theory. Our aim in this paper has been to point out that a withdrawal from equilibrium theorizing — which seems contradictory both to traditional Austrian thought in general and to Hayek's theory in particular — is not justified by the inability of pure economic theory to deal with radical ignorance. A critical, but positive, attitude towards the attempts to formalize radical ignorance suggests that the Austrian tradition may actually influence future research rather than merely constitute an optional supplement to it. As we have argued, the kind of formal representation of decision making under uncertainty one finds in recent developments in microeconomic theory is not intended to describe agents 'striving to formulate the correct vision of the future as if the future were something already implicit in the data and one's only problem is to guess correctly what the future will be' (Vaughn: 147). On the contrary, it recognizes as a starting point for research the view that ignorance is an inherent feature of every decision regarding future events. In this, it resembles the Shackleian assertion that the future is the unpredictable consequence of creative choices made by individual agents. And it seems to point towards a re-elaboration of the notion of equilibrium which is compatible at least with Hayek's, if not with the whole Austrian, tradition.

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